Interactive poster

Understanding and evolving precessing black hole binaries

Richard O'Shaughnessy

for
D. Gerosa, M. Kesden, E. Berti, U. Sperhake PRL in press [arxiv:1411.0674]
D. Trifiro, T. Littenberg, and above in preparation

Click here to begin… **or navigate as usual**
Use the script (paper) or listen to voiceover

Aspen Jan 2015, Black holes in Dense Star Clusters

Students

Davide Gerosa

Daniele Trifiro

What do you want to know?

- **Solving the binary black hole precession equations**
  - What are we solving and how?
  - Efficient solution for two-spin precession + inspiral
  - Understanding the adiabatic evolution of spin precession
  - Morphological classification

- **Parameter estimation**
  - Reasonable sources: identify signatures of both spins
  - “Morphological classification” and its astrophysical significance

- **Why should you care about precessing BH binaries?**

Binary inspiral and precession

$$\frac{dv}{dt} = \frac{32}{5}\frac{\eta}{M}v^9\left\{1 - v^2\frac{743+924\eta}{336} + \ldots + O(v^8)\right\}$$

$$\frac{d\mathbf{S_1}}{dt} = \mathbf{\Omega_1} \times \mathbf{S_1}$$

e.g, Kidder 1995; Apostolatos et al 1994

$$\frac{d\mathbf{S_2}}{dt} = \mathbf{\Omega_2} \times \mathbf{S_2}$$

$$\frac{d\mathbf{L}}{dt} = \mathbf{\Omega_L} \times \mathbf{L} + \frac{dL}{dt}\hat{L}$$

- Orbit, precession, inspiral timescale hierarchy
  - Conserved on precession time $\mathbf{J} = \mathbf{L} + \mathbf{S}_1 + \mathbf{S}_2$
  - Conserved on precession time and (2PN) adiabatic invariant $\xi \equiv \frac{\mathbf{S_0} \cdot \hat{\mathbf{L}}}{M^2}$

Constant from Racine 2008

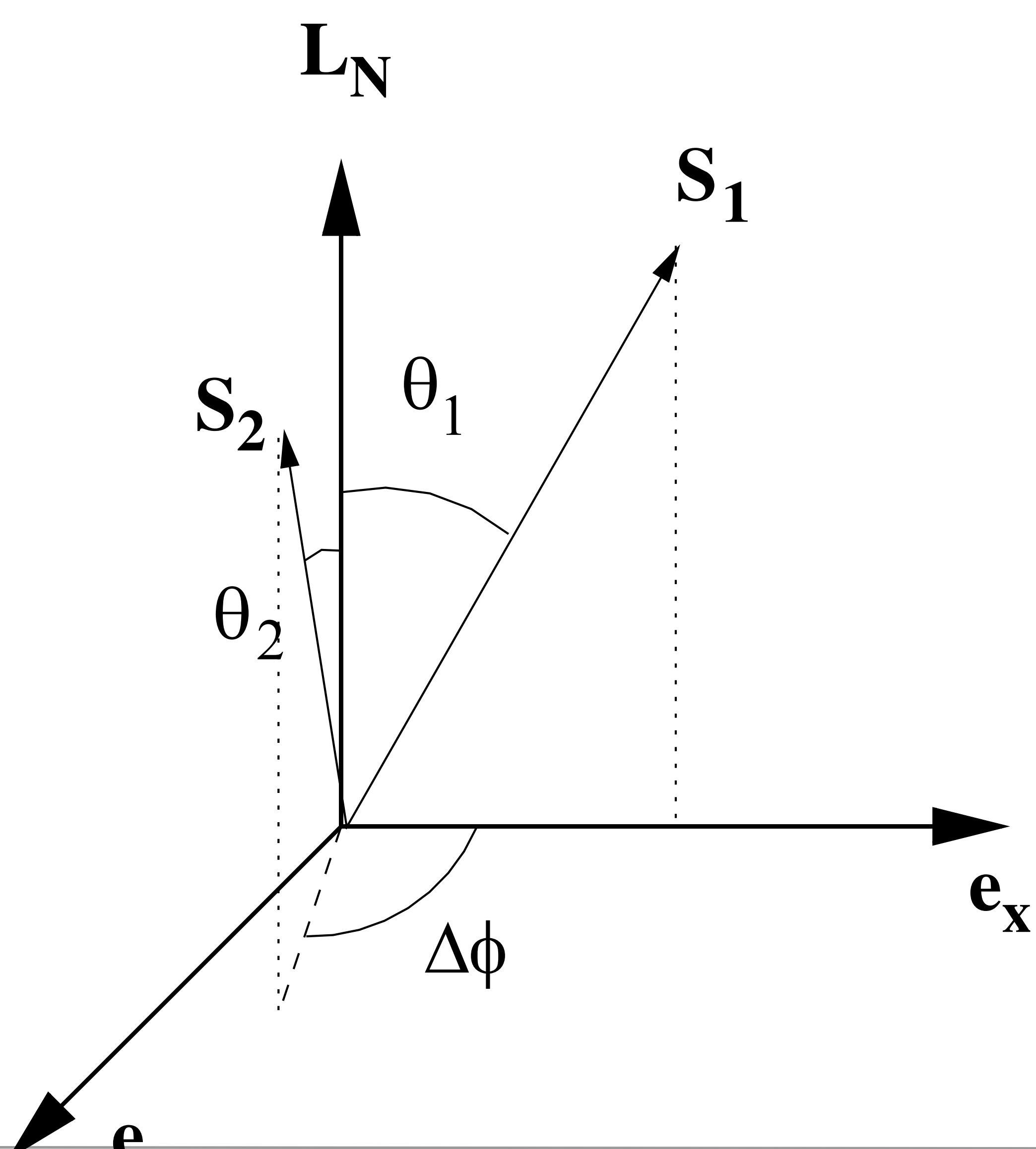
$$\mathbf{S_0} = (1+q)\mathbf{S_1} + \left(1 + \frac{1}{q}\right)\mathbf{S_2}$$

Coordinates for precessing spins

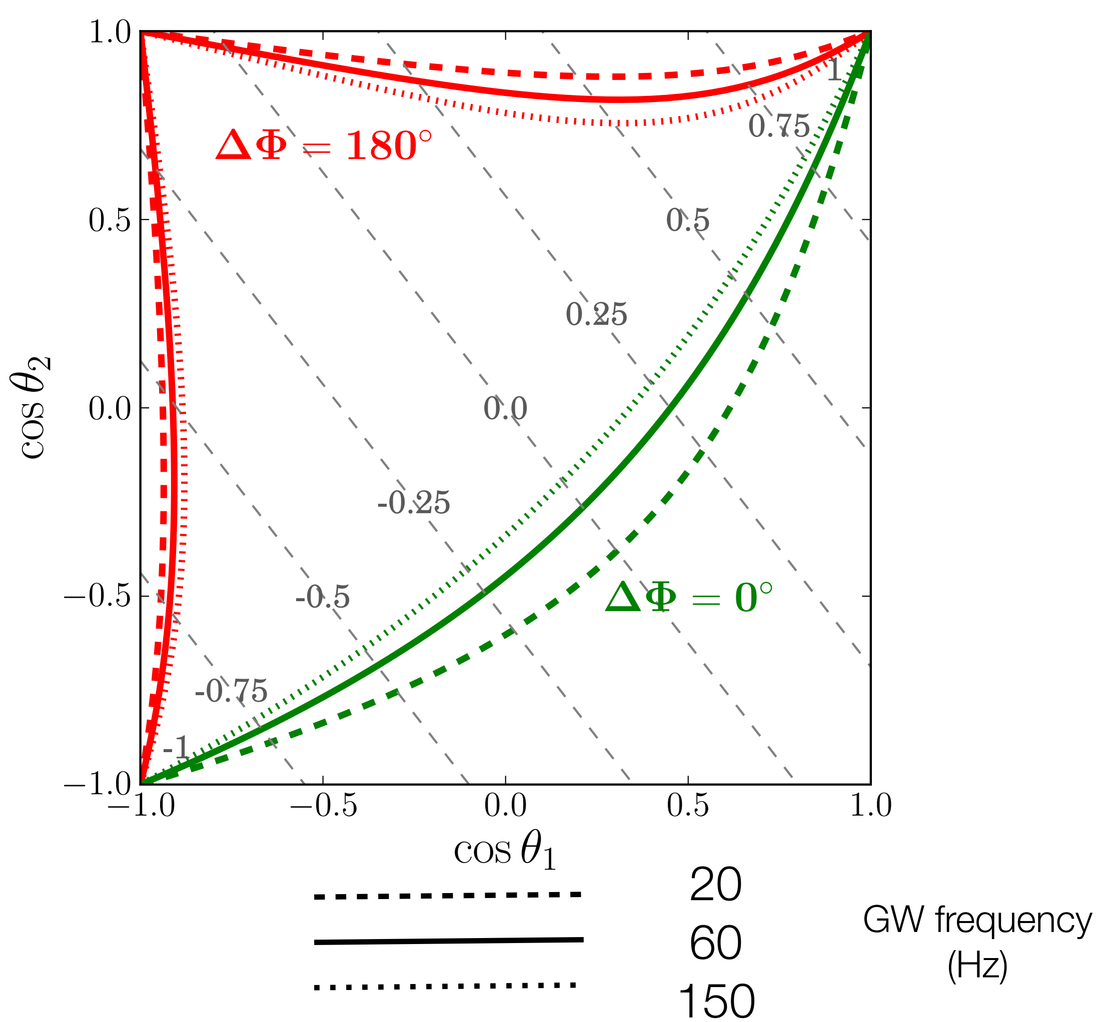
- Spin vectors relative to L

$$\cos\theta_1 = \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{L}}, \qquad \cos\theta_2 = \hat{\mathbf{S}}_2 \cdot \hat{\mathbf{L}},$$

$$\cos\theta_{12} = \hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2, \quad \cos\Delta\Phi = \frac{\hat{\mathbf{S}}_1 \times \hat{\mathbf{L}}}{|\hat{\mathbf{S}}_1 \times \hat{\mathbf{L}}|} \cdot \frac{\hat{\mathbf{S}}_2 \times \hat{\mathbf{L}}}{|\hat{\mathbf{S}}_2 \times \hat{\mathbf{L}}|}.$$



- Separate problem
  - Solve “co-precessing frame”: spins relative to L (3-d ODE)
  - Integration solves inertial frame



Solving precession

- **Conservative evolution** (L fixed) $L \leftrightarrow r$
  - Count parameters: only one left!
    - $\mathbf{L}, \mathbf{S}_1, \mathbf{S}_2$ magnitudes conserved 6 (directions only)
    - Frame aligned with L 3 (and S1 in 'xz' plane)
    - Magnitude of J conserved 2
    - ξ conserved 1
  - Recover inertial frame L, spins by integration + rotation

$$\frac{d\mathbf{L}}{dt} = \mathbf{\Omega}_L \times \mathbf{L}$$

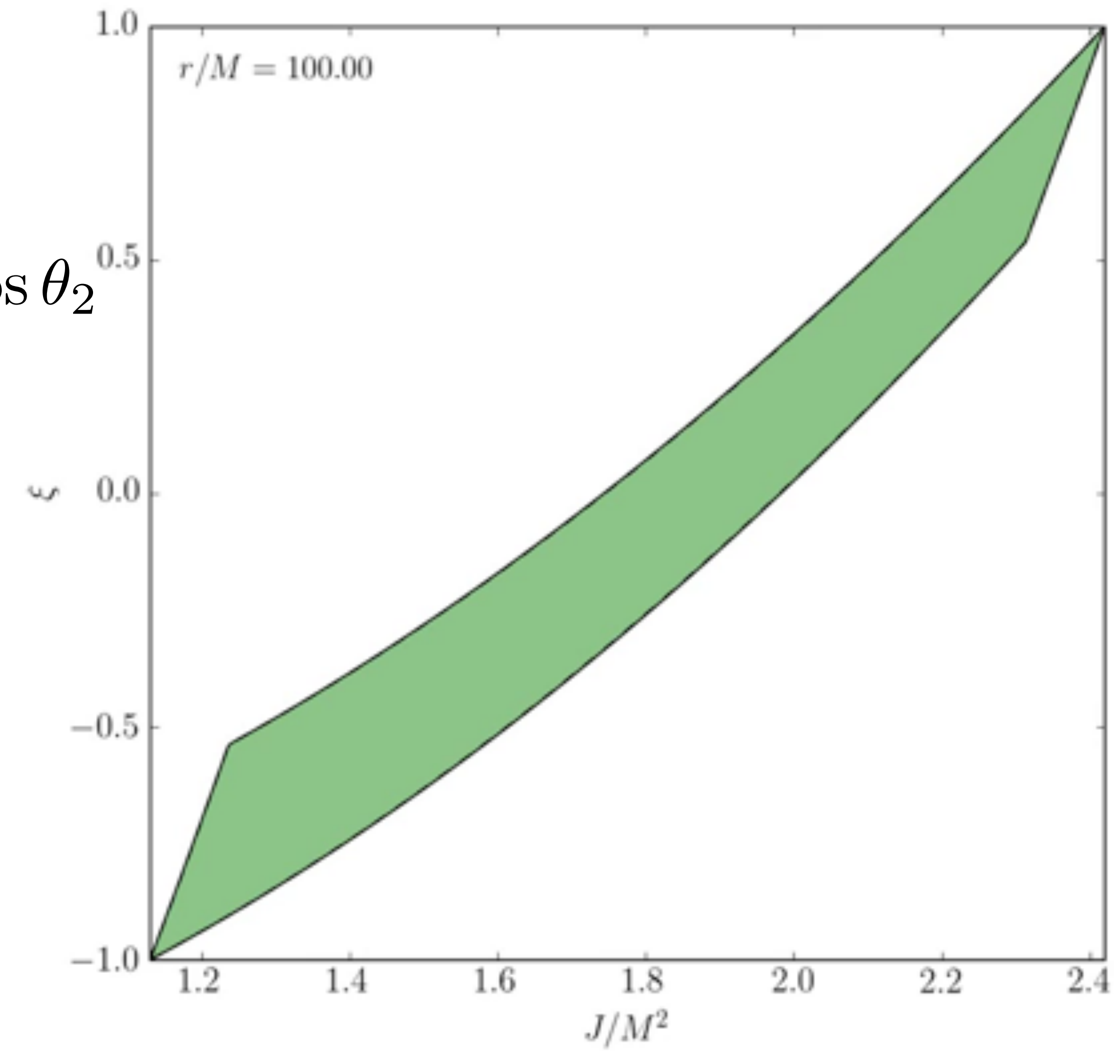
- **Precession and inspiral**
  - Because L precesses around J
  - Average is one precession cycle

$$\frac{dJ}{dL} = \left\langle \hat{\mathbf{J}} \cdot \hat{\mathbf{L}} \right\rangle$$

Other approaches: Montana State 2013; Tessmer et al 2013;

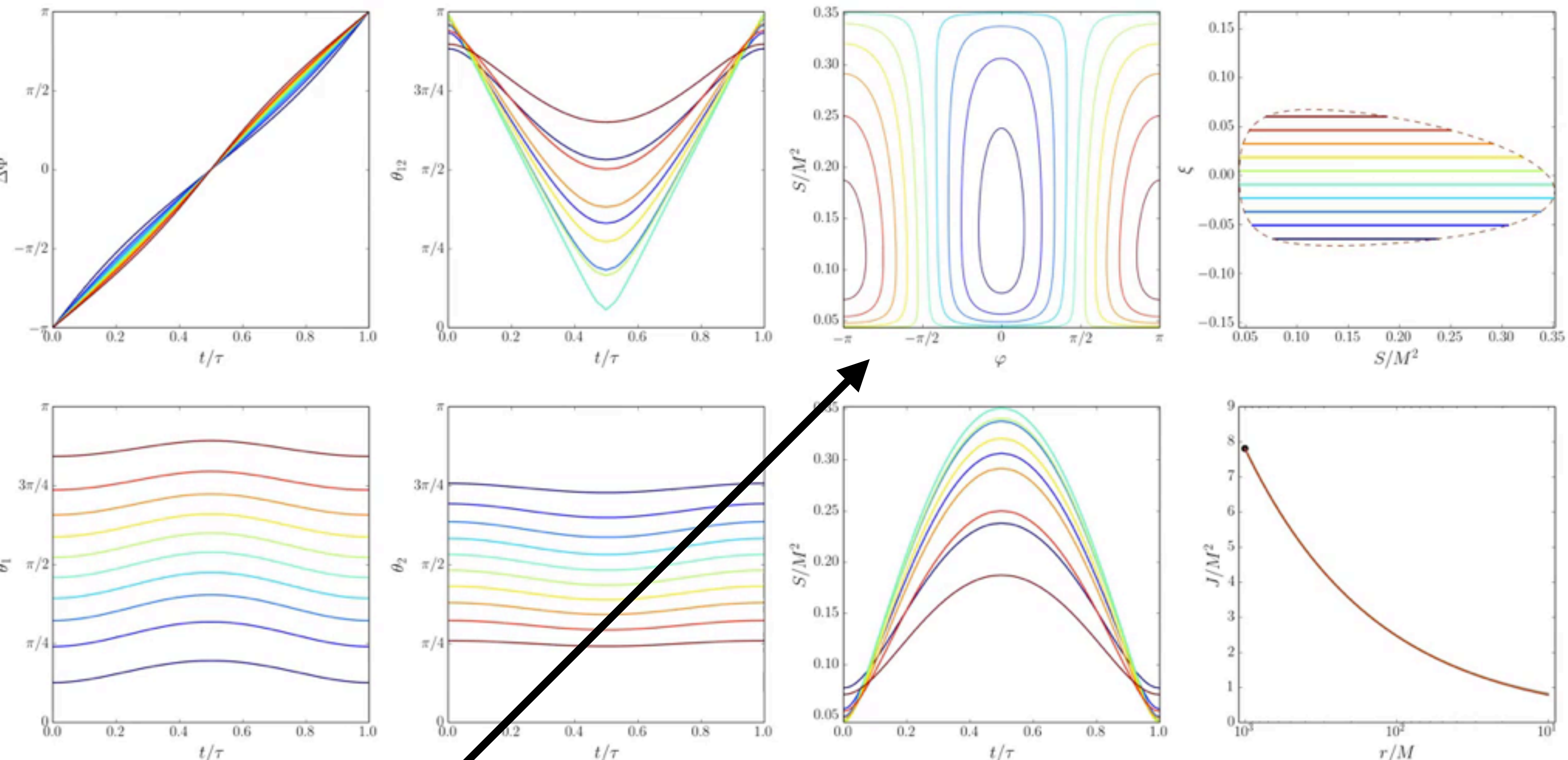
Solution enhances computation and insight

- **Efficient**
  - Minimal stepping (=inspiral timescale)
  - Can evolve astrophysical scales according to theory (not randomly), *cheaply*
- **Evolution in** J, ξ
  - Conserved constants as coordinates
    $$\mathbf{J}^2 = \mathbf{L}^2 + (\mathbf{S}_1 + \mathbf{S}_2)^2 + LS_1 \cos\theta_1 + LS_2 \cos\theta_2$$
  - Movie: allowed region vs time
  - Evolution to low J with “similar” shape
    - At large r, J, ξ allowed region = “distorted” $\cos\theta_1, \cos\theta_2$ plane



q=0.3, maximal spins

How do spin dynamics change versus time?

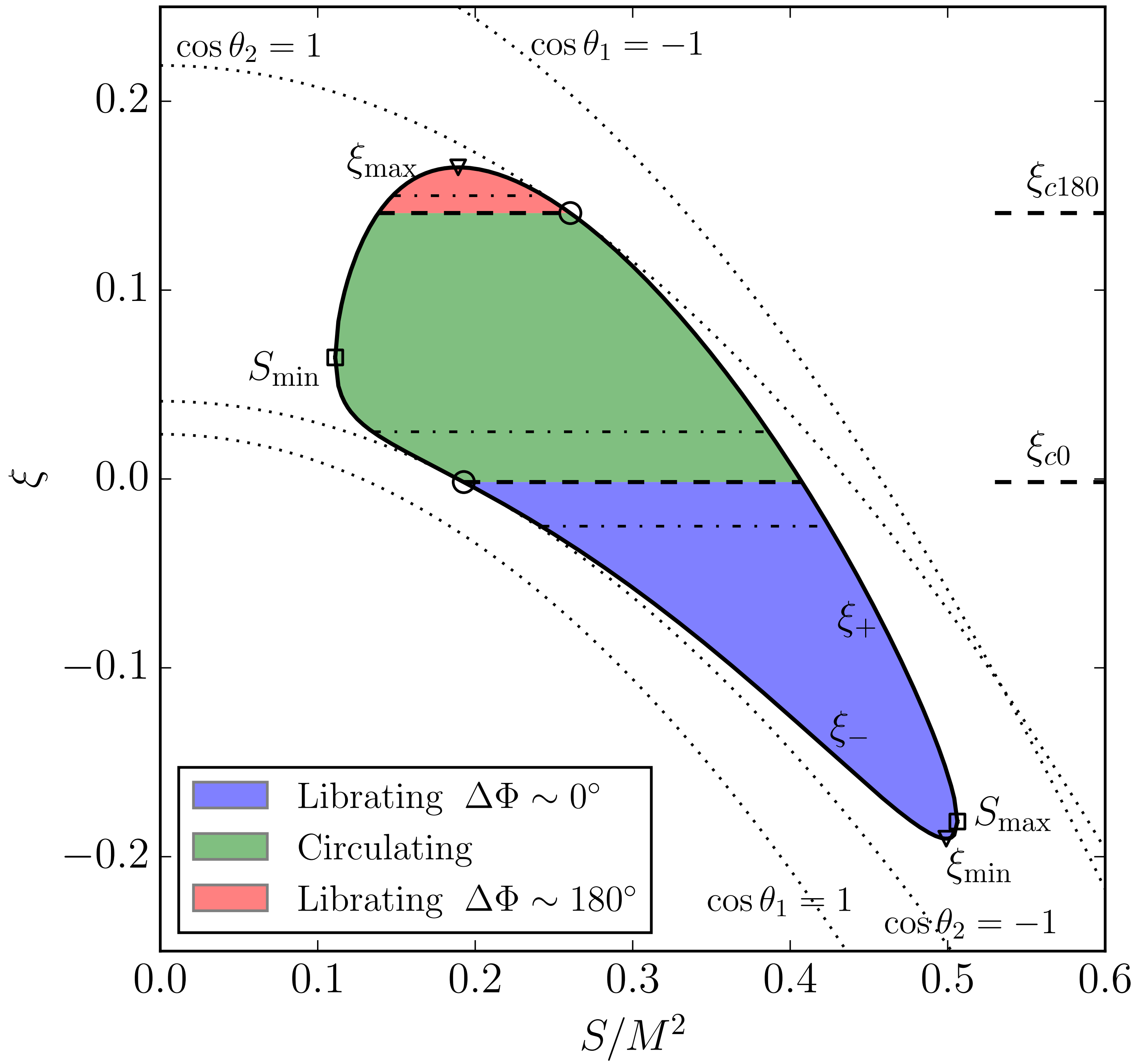


$$S = |\vec{S}_1 + \vec{S}_2|$$
$$\varphi' = \text{azimuthal angle of } \vec{S}_1 \text{ about } \vec{S}$$

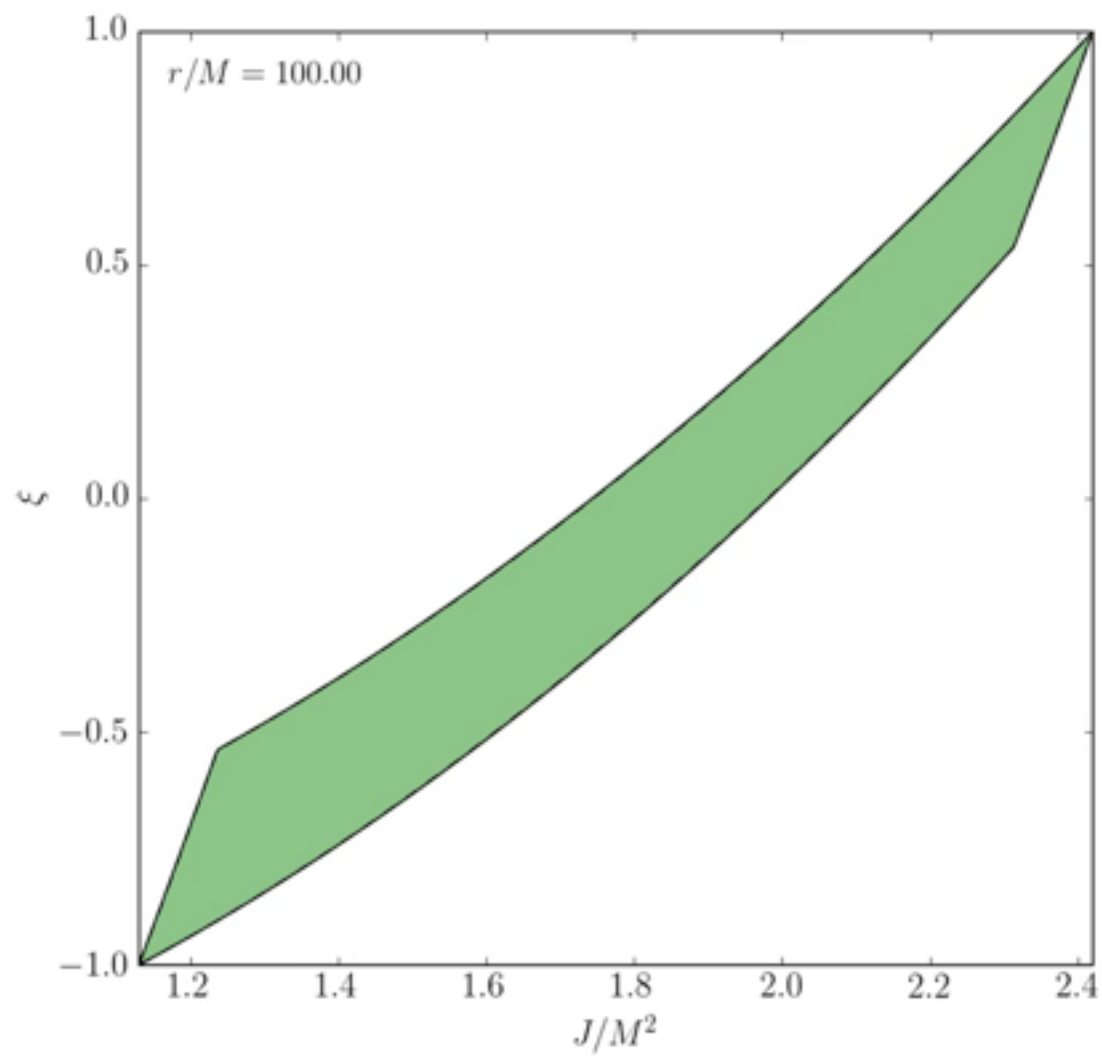
Inspiral

Phase space for double-spin evolution

- **Fixed points:** Global maximum, minimum of ξ at fixed J

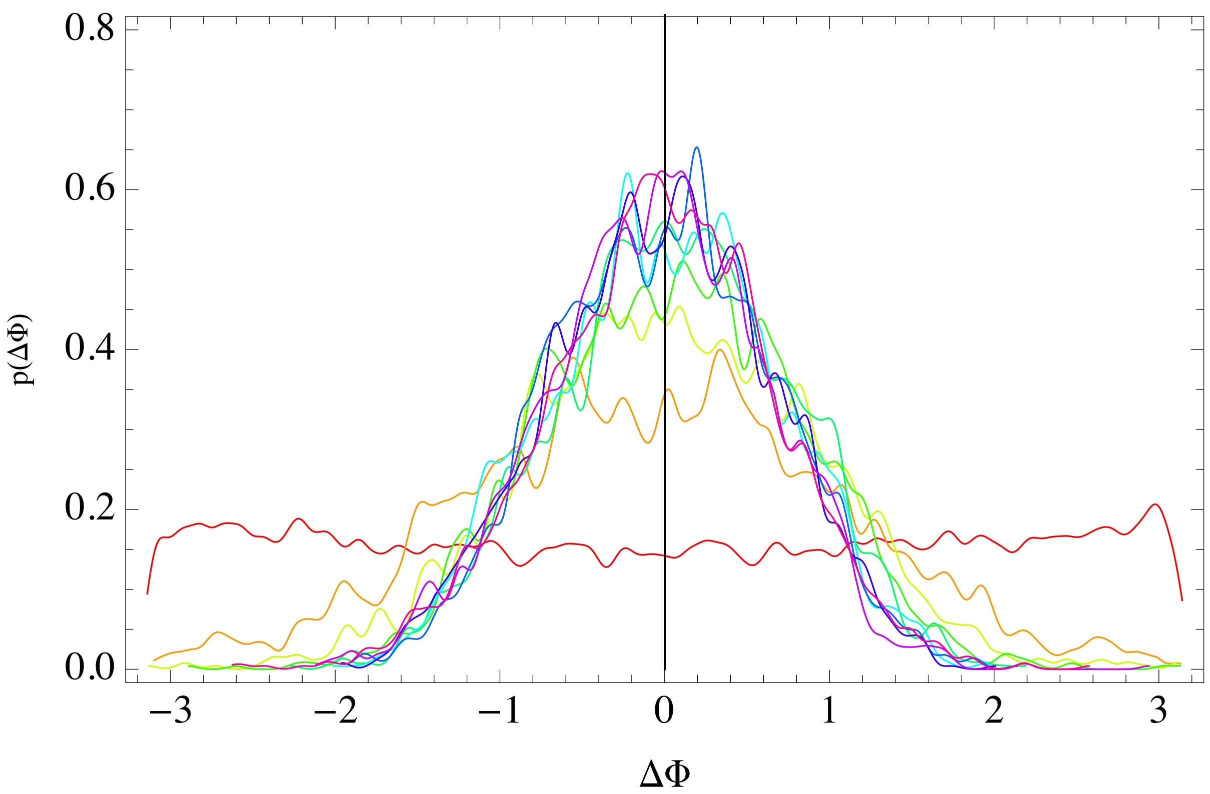


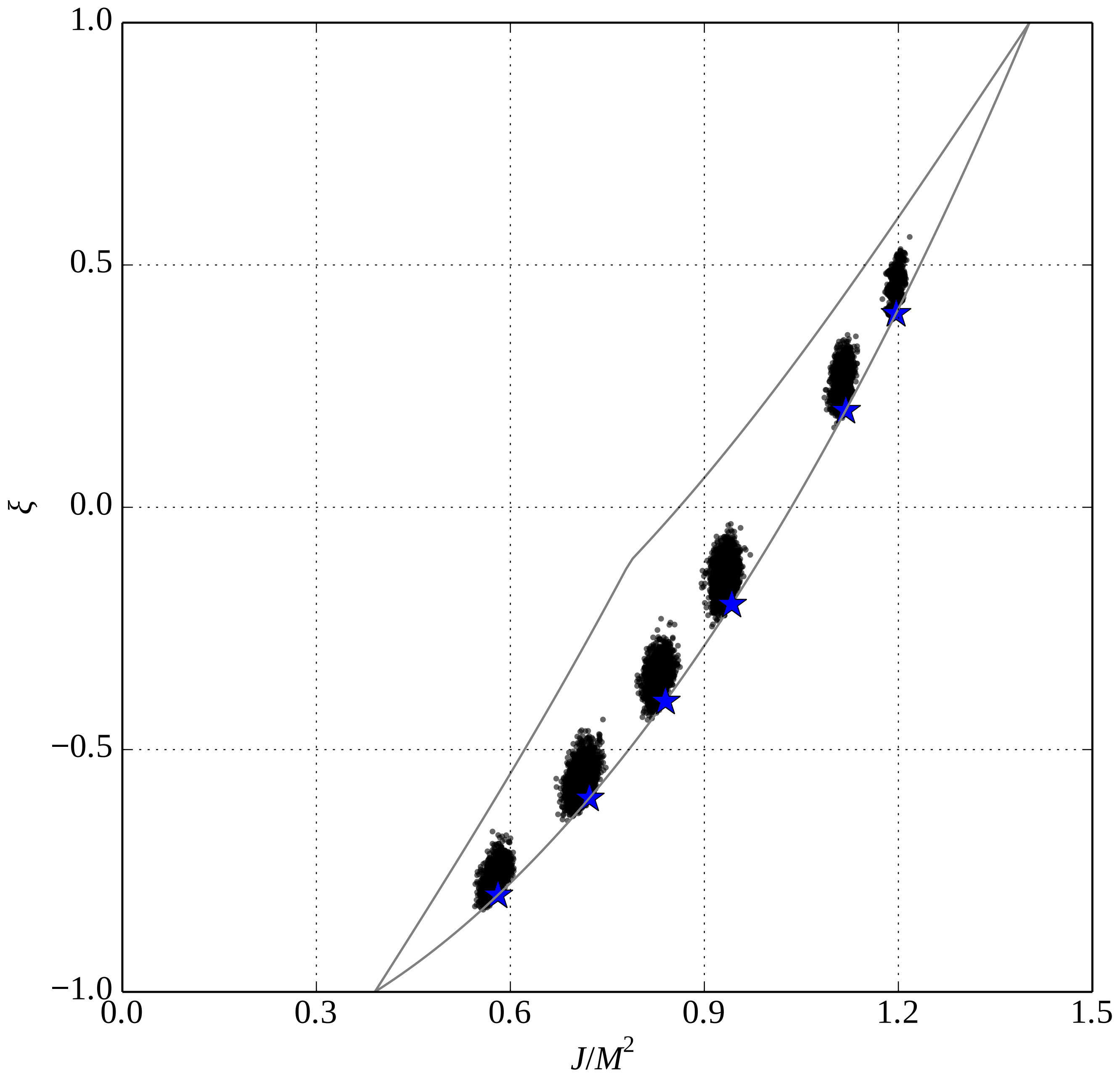
“Transfer function” and precession morphology



Parameter estimation: Parameter constraints

- GW measurements constrain plausible source parameters
- **Example**: Pick **exactly** resonant sources (=edges of J, ξ phase space)

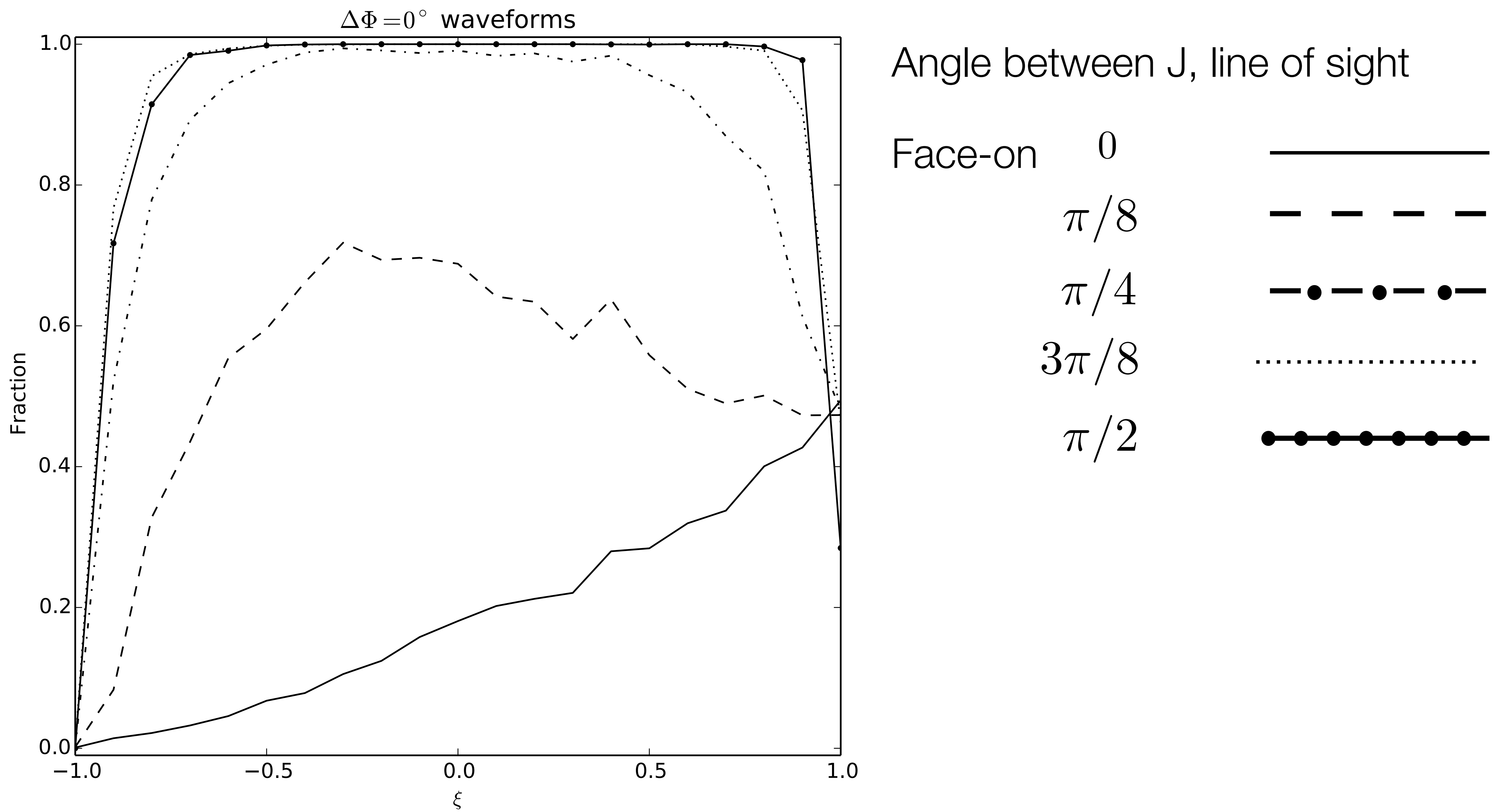




- Clear signature of **both** spins

Parameter estimation: Distinguishing resonances

- Correctly identify "morphology" …. except "face-on", non-modulated binaries

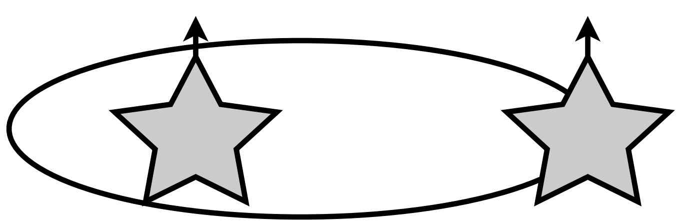


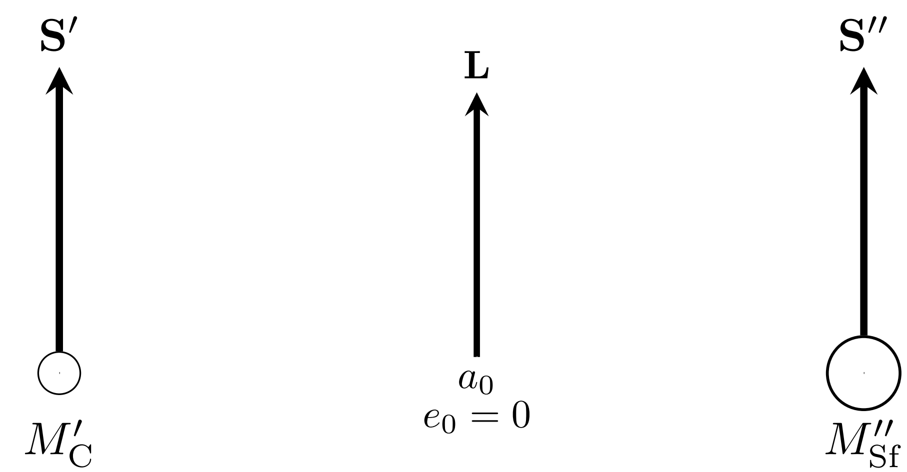
How might BH-BH binaries form?

- **Formation order encoded in morphology?**

Inertial frame

In frame of orbit





Gerosa et al 2013

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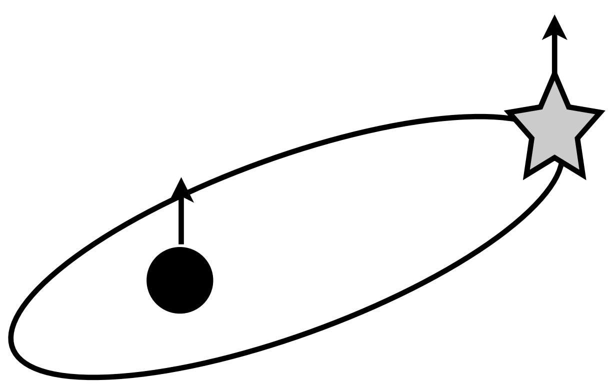
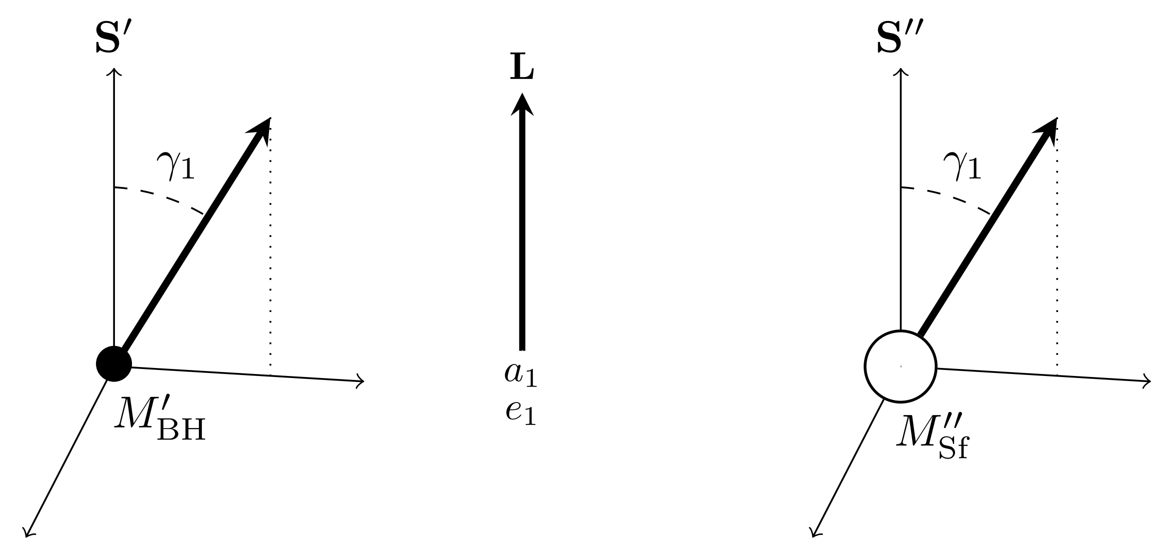


In frame of orbit



Gerosa et al 2013

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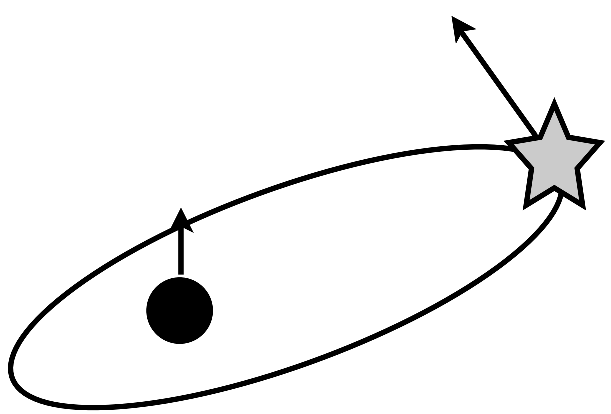

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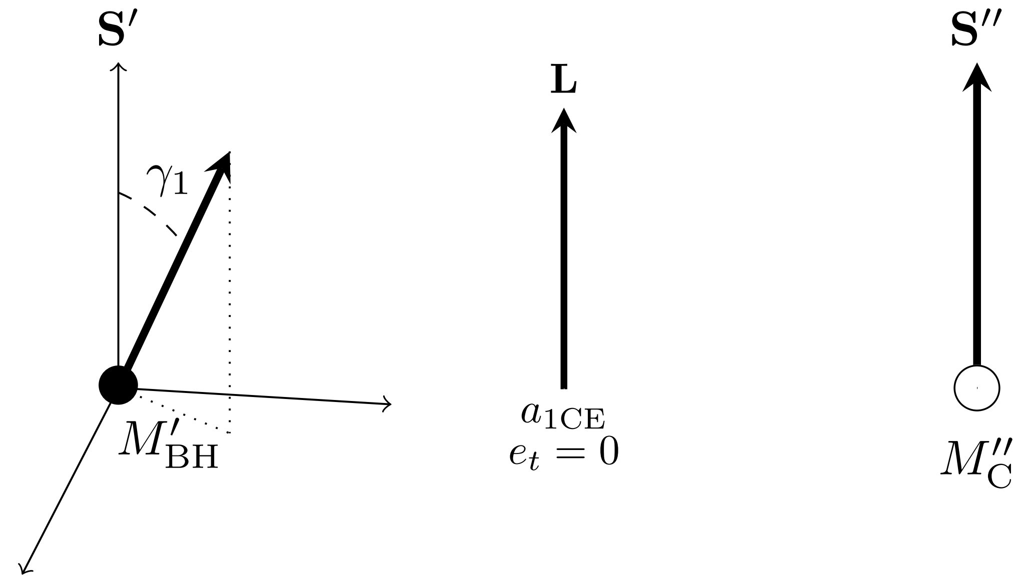

Gerosa et al 2013

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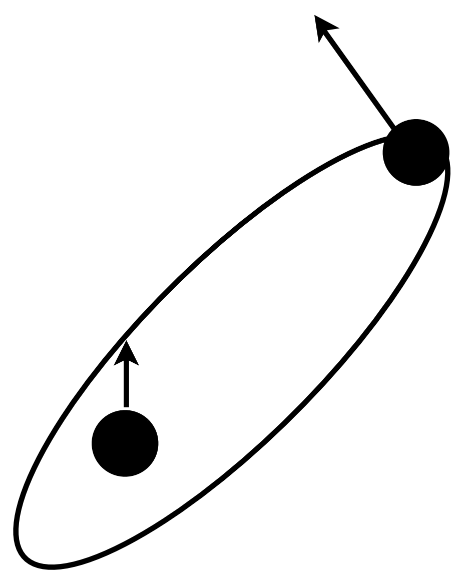
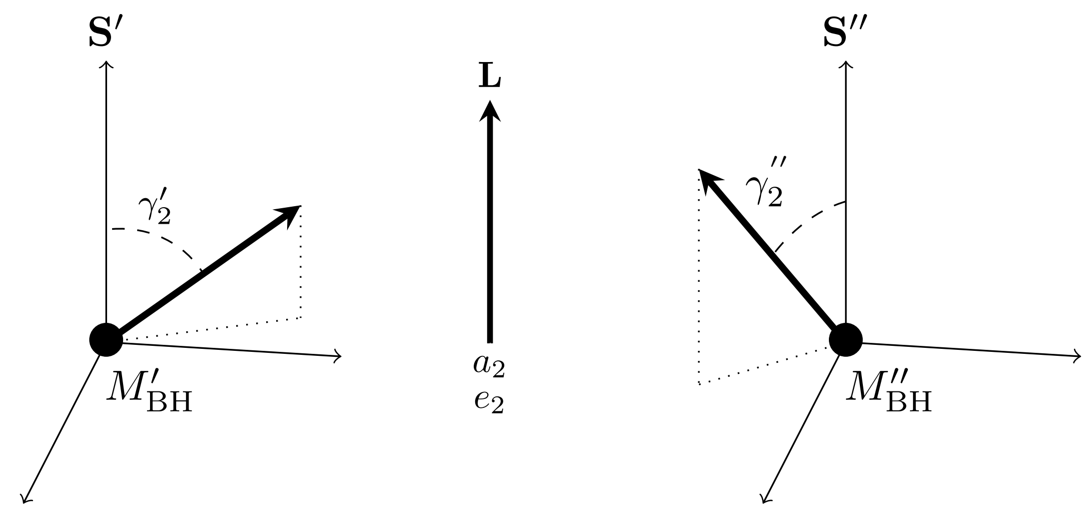


In frame of orbit



- **Key feature:**
  - first-born BH has larger misalignment [need not be most massive]
  - distinguishable gravitational waves, via **morphology**

Astrophysical motivation

- **Supermassive black hole evolution over time**
  - Recoil
  - Merger dynamics and EM signals

Talk Sunday: Blecha
Talks Sunday: Bogdanovic, Krolik, Rasskazov, Noble,MacFadyen

- **Gravitational waves from merging binaries**
  - Relating asymptotic ("birth") and near-merger ("observable") spins
  - Inferring binary parameters from detected gravitational waves
  - Insight into formation processes: SN kicks, birth order,…

2015-01 Aspen Interactive Presentation Script
Understanding and evolving precessing black hole binaries

R. O'Shaughnessy

Binary black holes (BBHs) on quasicircular orbits are fully characterized by their total mass M, mass ratio q, spins $\mathbf{S}_1$ and $\mathbf{S}_2$, and orbital angular momentum $\mathbf{L}$. When the binary separation $r \gg GM/c^2$, the timescale on which these angular momenta precess is much shorter than the radiation-reaction time on which $L = |\mathbf{L}|$ decreases due to gravitational-wave (GW) emission. Using conserved constants and separation of timescales, we solve the orbit-averaged spin-precession equations analytically for arbitrary mass ratios and spins. This solution provides a simple and efficient way to propagate binaries at astrophysically interesting separations (e.g., $10^6 M$) to just prior to merger, enabling further studies of SMBH evolution and recoil. The solution decomposes BBH spin precession into three distinct morphologies, between which BBHs can transition during their inspiral. This solution also provides new insight into the complex, nonlinear, two-spin dynamics of binary black holes and the gravitational waves they emit, enabling us to better understand and interpret gravitational waves from generic BBH mergers and to exploit those results for astrophysics.

Where to find more information:
Paper: http://arxiv.org/abs/1411.0674
Movies

```
https://www.youtube.com/watch?v=rKET4753MfE   : single evolution of J, \xi plane inwards
https://www.youtube.com/watch?v=FGYkEvJ8z4s&list=PLwaaLoSFA-EcfUJ-k1rviZBs-Ct-ze9Tk&index=3:
```

PDF slides: http://ccrg.rit.edu/ oshaughn/2015-01-Aspen-Interactive.pdf
PDF script: http://ccrg.rit.edu/ oshaughn/2015-01-Aspen-Interactive-Script.pdf

I. SCRIPT

```
1: Introduction
    Physical Review Letters has just accepted work that my collaborators and I have  finished,
    giving a new perspective on an old problem: the impact of spin on the
    orbit and inspiral of black hole-black hole binary.  In this interactive presentation, I'll describe
    our solution and why you, an astronomer or astrophysicist at Aspen, should care.


2: Collaborators
    This work obviously wouldn't be possible without the hard work of my collaborators, notably Davide Gerosa from
    Cambridge  and Daniele Trifiro from Pisa, who worked on the general theory of double spin evolution and
    on gravitational wave parameter estimation, respectively.



3: Outline
    Please feel free to navigate this interactive presentation as you see fit,
    using the navigation links on the bottom of each slide and within selected slides,
    including this outline.

    Elements of this presentation describe

    - Solving the binary black hole precession equations
      - What the precession equations are, and how we solve them.
      - Fast and insightful inspiral calculations with two spins
      - A transfer function: How precessing spins periodic dynamics change with time
      - "Morphological classification": The spin phase space split into three parts, which
        precess in distinctive ways and which may be populated by astrophysicall distinct properties.

    - Inferring parameters of resonant binary black holes
```

- Why you should care and, qualitatively, what our astrophysics solution facilitates

4: Mathematics of inspiral
The inspiral and precession of binary black holes are well-characterized by the following post-Newtonian equations.

- The first equation shows how the orbit of a binary of total mass M and reduced mass \eta M shrinks, as characterized by its velocity 'v'.
  The first term is the Peters and Mathews equation.

- The second equations describe how the spins S1, S2 precess. Spin precession occurs because of spin-orbit coupling and spin-spin interactions, hidden inside \Omega_{1,2}

- The evolution equation for L is also a precession equation, plus a term involving GW radiation losses. The spin precession rate for L is fixed by requiring conservation of J (if dissipation was removed.)

A strong timescale hierarchy exists: the orbital period is much shorter than the precession timescales (1/\Omega_{1,2,L}), which is in turn much smaller than the inspiral time (v/(dv/dt)).

Dissipation only occurs on the inspiral time; on shorter timescales, the spin evolution is conservative, well-described by a hamiltonian. This hamiltonian evolution conserves the total angular momentum J and (at 2PN order) a special linear combination of the spins, \xi. On longer timescales, J shrinks in magnitude but \xi remains nearly constant, being an adiabatic invariant.

Double-spin dynamics has historically been studied numerically.
These solutions show L, S1, and S2 precess around the direction of J, which remains nearly fixed.

Several people have presented different semianalytic solutions, notably Racine 2008.
They are complicated or incomplete -- for example,they work for perturbatively small spins; only in the equal mass case; etc.

5: Coordinates for inspiral in a special, co-precessing frame
To characterize the two spins' evolution and solve the EOMs, we need to define several variables, which describe the evolution of the spins in a frame aligned with L. Specifically, in this frame L is along the z axis and S1 is in the x-z plane.

Note that in this frame, the problem only has *3* parameters: \theta_1, \theta_2, \Delta\Phi.

6: How we solve for precession and inspiral
The conservative evolution, appropriate to short timescales and nearly fixed separation r or L can be solved in the frame aligned with L using known conserved constants: the problem reduces to one-dimensional hamiltonian evolution.
Having solved the *relative* motion ODEs, we have the information needed to solve dL/dt and construct the spin trajectories in an inertial frame.

Because L precesses around J and because the binary instantaneously radiates angular momentum along L, the "average" rate of total angular momentum loss is along J, with a coefficient set by the average angle between L and J. Using the spin trajectories, this precession-time average can be evaluated.
So we can compute how J(L). Using \xi constant, the long-term spin evolution is completely specified.

7: Useful computationally and conceptually
This solution is computationally vastly more efficient than previous numerical approaches to explicitly solve the spin precession equations. Though some numerical evaluation is required, each timestep can be a significant fraction of the *inspiral* timescale. We can therefore efficiently evolve binaries from astrophysically relevant separations down to merger, including stellar-mass binaries formed in few hour orbits and supermassive black hole binaries.

Our solution also provides insight into had previously been a conceptually opaque problem with a few tantalizing results, like post-Newtonian resonances. As an example, our analysis suggests one natural way to represent inspiral is the almost-trivial "flow" of binaries in the J, \xi plane, shown in the movie. At each time, the movie shows *all* allowed J, \xi combinations for the specific masses and spin magnitudes.

This representation has several advantages:

- Phase angles suppressed: Since J and \xi specify the spin precession evolution, uniquely identifying a phase-space trajectory for spins at each time. This plane suppresses trivial phase angles that change on the precession time but are needed to specify the precise orbital angular momentum and spin directions.

- Easy to interpret at large distances: At large separations (large L), the "distorted parallelogram" has a trivial 1-1 relationship with the \cos \theta_1, \cos \theta_2 plane

- Evolution "boring": Because \xi is conserved and J decreases (until and past J=0 configurations), binaries evolve trivially to the left in this plane.

- Edges and features have meaning: Finally, as we will see shortly, the edges of the J, \xi plane correspond to special configurations: "post-Newonian resonances", where the spins and L remain coplanar while they precess.

We will return to the colors shown in this movie momentarily.

8: Understanding spin precession: phase trajectories at each separation, and how those trajectories evolve
Because the J,\xi plane explicitly suppresses precession, we need to use other variables to illustrate the phase-space trajectories of the spins, and how those trajectories change over the inspiral.

This movie shows the evolution of binaries with fixed J, but a color-coded range of \xi values. By construction, each J,\xi combination corresponds to a unique phase-space trajectory at each separation. On the precession time, each binary evolves periodically along the color-coded contours.

In the movie's top 3rd panel, we use two coordinates to describe the the relative spin evolution at fixed J in the 3d spin space (e.g., \theta_1,\theta_2, \Delta \Phi, restricted to fixed J). The closed contours clearly include two fixed points. The fixed points correspond to the largest and smallest values of \xi allowed, at each J -- in other words, the edges seen in the J,\xi plane in the previous slide. More broadly, the contours either circulate around one or the other maximum, or circulate between them.
As time evolves, the binary separation shrinks and the isocontours evolve, usually adiabatically.

Now look at the top left and bottom right panel: \Delta \Phi (the angle between the spins) and \theta_1,2, both plotted versus the precession time, showing how these quantities will periodically evolve each precession cycle. Early on the relative spin angles all circulate through the full range of angles. For some of the contours, eventually one of the two spins becomes aligned with L (\theta_{1} or \theta_2 ->0,\pi When this occurs, the evolution in \Delta\Phi undergoes a *transition* -- in this movie, the binaries illustrate become "trapped" near \Delta\Phi =0, or coplanar spins.

9: Morphological classification
To start with, what does it mean for a binary to be trapped, with hamiltonian evolution circulating \Delta\Phi=0 or \pi, or not? For each J, \xi, what calculation tells us which of the options, o or "morphologies", that contours of constant \xi correspond to?

This figure shows the allowed region in the \xi, S plane at fixed 'J'. As the spins precess, a binary evolves periodically along lines of constant \xi, with S changing periodically. Each allowed point in this plane corresponds to a unique value of \Delta\Phi, \theta_1, and \theta_2; contours correspoding to one spin being aligned (\cos \theta_1 =\pm 1 and \cos \theta_2 = \pm 1) are shown.

Note the following:

- A unique maximum and minimum exists, corresponding to resonant fixed points.
Mapping back to the J,\xi plane, the 'edges' of the allowed region in J,\xi are all resonant.

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    - \Delta \Phi must be either 0 or \pi on the boundary

    - \Delta \Phi must be continuous on the boundary, with the option of changing discontinuously
      *only* when one spin is aligned or antialigned (and hence \Delta\Phi, the angle between the spins,
      is ill-defined).

   Using these ideas, with careful analysis, one can show that for sufficiently large J, the phase
   space breaks up into three parts.

    - In the red region, above xi_{c,180}, \Delta\Phi=\pi on both the left and right edge
    - In the green region, between \xi_{c,0} and \xi_{c,180}, \Delta\Phi=0 on the left and \pi on the right.
    - In the blue region, below \xi_{c,0}, \Delta\Phi=0

   Accounting for subtleties, this rule can be generalized to small J


10: Morphological classification: J, \xi plane
   Using this rule, we color the state of each point in the J, \xi plane.

   Note the following:
    - Resonances at fixed \xi:
      At *fixed \xi*, binaries with the smallest J are in the \Delta\Phi=180 resonance (red);
      the largest J are in the \Delta\Phi=0 resonance (blue); and the rest are circulating.

   - Resonances are ubiquitous:
      A *significant* fraction of binary phase space evolves into being "trapped" near these
      resonances.


11: Gravitational wave parameter estimation
   Merging binaries emit gravitational waves which completely encode their dynamics and properties.
   By systematically comparing observations with models for these generic, highly-modulated signals,
   we can infer all properties of the binary.

   Some properties, however, are easier to discern than others.

   -  The rapid increase in orbital and gravitational wave frequency encodes the masses and (aligned)
      spins.

   -  Modulations in the gravitational wave signal traces the orientation of the orbital plane relative
      to the line of sight -- in other words, the precession rate (and opening angle) of L as it evolves
      around J.

   -  Because black hole spin scales like mass^2, and because the relative spin orientation has
      a subdominant effect, in many cases the precise spin magnitude and direction of
      the smaller body is difficult to constrain.  This **suggested** that double-spin effects were
      hard to measure.

   To assess this hypothesis, we create synthetic data for "exactly resonant" sources -- in the J, \xi
   plane, binaries on the edge of the allowed region -- for a range of source orientations relative
   to the line of sight.  We then infer what source parameters could be consistent with our synthetic data,
   making *no assumptions* about the source.

   These two plots show results of this analysis, for one specific degree of alignment between
   the source and the line of sight.  In these figures, binaries are characterized by their orbital
   properties when f_{gw}=100 Hz.

   - Left plot:
     On the left, I plot the posterior density for \Delta\Phi for sources with different \xi.
     In short, except for \xi =-1 (=aligned spins), the relative angle between the spins can be measured.
     In other words, we can explicitly confirm the signature of a second, small spin.

   - Right plot:
     On the right, I superpose
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      - the "allowed" region in the J,\xi plane for the true source [solid line]
      - a sequence of injected resonant source (J,\xi) points [stars]
      - our MCMC estimate of the best-fitting J,\xi parameters.
        Note these include sources with generic masses, spin magnitudes, et cetera.
        GW measurements well-identify both natural parameters of these double-spin systems.


   These results suggest we can tell if a binary is circulating about one or the other resonance,
   and measure its characteristic precession parameters or timescales well.




12: Morphological classification
   A significant fraction of evolving binaries move into the red or blue trapped region.  As we discuss below,
   astrophysics may preferentially produce binaries near the top or bottom edge of the J, \xi plane, corresponding
   to preferential alignment of the primary or secondary.  So we care not only about the binary *parameter*
   distribution, but whether it is 'red, blue or green'

   We have applied our classification rule to our posterior, properly accounting for varying mass and spin in
   candidate sources.   For the "bottom edge" resonant binaries (\Delta\Phi=0), this plot shows the
   fraction we *correctly* classify (as blue), for
     - different angles between the line of sight to the source, and
     - different values of \xi

   Except for face-on systems or nearly-aligned systems (extremal \xi = -1,1), we can reliably identify the
   source morphology
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